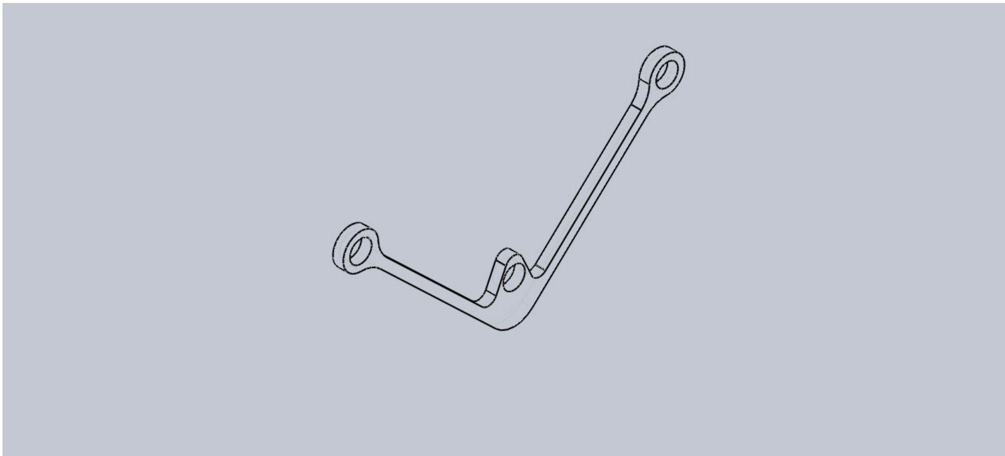
David Oke
Id: doke

Project 1
Main Report

2. Summary



Description:
a V-shaped bracket angled at approximately 45 degrees. The arm widths on the bracket were minimized to circumnavigate the forbidden zone allowing for greater vertical loading.

Reasoning:

The main reason for my design was to limit stresses due to bending and to create as much axial stress as possible. As a former student of Professor Steif's class of Stress Analysis, I've come to recognize axial stress as the best possible stress. My design as a result is meant to circumnavigate the geometric restrictions (i.e forbidden zone) while also producing uniformly axial stress across the bracket.

Part Estimations:

Mass: Density * Volume = (1.19*1,000)*.147 * 10^{-6} m^3= 1.75 grams
Factor of Safety: 4.5-5
Failure Load: 120
Prediction of Failure Mode: Bending failure at fillet intersection of bracket arm and peg hole.

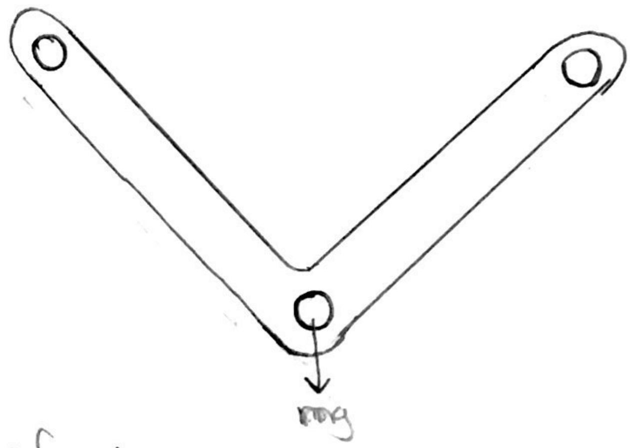
3. Simple Models, Free Body Diagrams, and Simple Failure Analysis

(See Next Page)

David Ohe

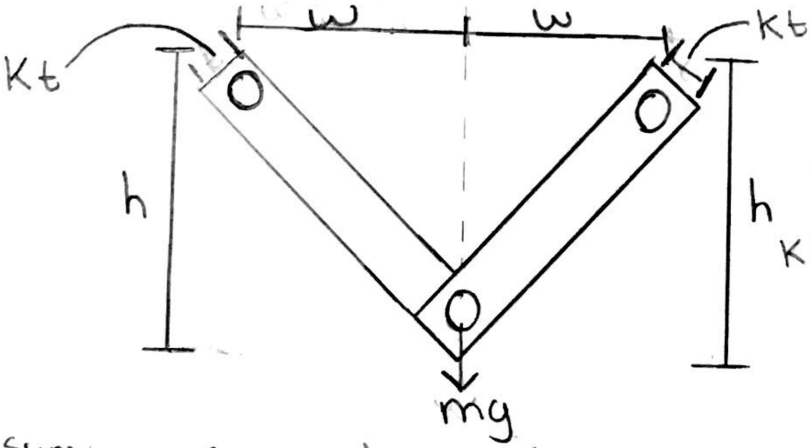
3.

Proposed Design



a) Simplification of design:

- Treat part as two overlapping rectangles connected at the support clip.

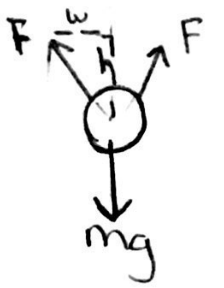


Parameters
w= width of one rect.
h = height of one rect.
Kt= Cross-sectional area

- Assuming symmetrical loading on rectangles, I only need to model one rectangle with a load of $\frac{mg}{2}$

b) continued

FBD of Pin



(assuming the rectangles are two-force members and are symmetrical)

$$\cos\theta = \frac{h}{\sqrt{w^2+h^2}}$$

$$\Sigma F_y \rightarrow 0 = -mg + 2F\cos\theta$$

$$mg = 2F\cos\theta$$

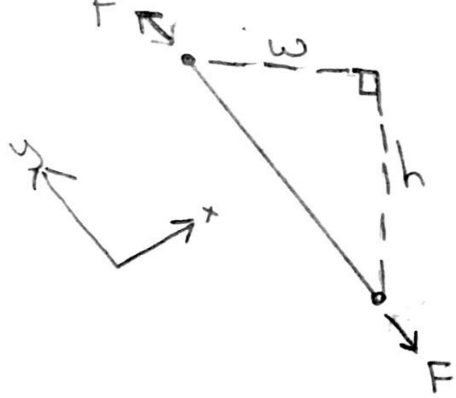
$$\frac{mg}{2\cos\theta} = F$$

$$\frac{mg}{2}\cdot\frac{\sqrt{w^2+h^2}}{h} = F$$

$$\frac{mg}{2}\cdot\sqrt{\frac{w^2+h^2}{h^2}} = F$$

$$\boxed{\frac{mg}{2}\sqrt{1+\frac{w^2}{h^2}} = F}$$

FBD of left rectangle



(axial loading so no shear or bending stresses)

(Also assuming weight of part is negligible)

$$\Sigma M|_{\text{any point}} = 0$$

$$\Sigma F_x \rightarrow 0$$

$\Sigma F_y \rightarrow F = F$ (Two force member)

c) Possible stresses

Based on assumption that it's a two force member, Only stresses are axial stress & contact stress. No buckling because it's in tension and not compression

$\sigma_m = 0$

$\tau = 0$

$\sigma_A = \frac{F}{A}$ where $F = \frac{mg}{2}\sqrt{1+\frac{w^2}{h^2}}$, $A = Kt$

$$\sigma_A = \frac{mg\sqrt{1+\frac{w^2}{h^2}}}{2Kt}$$

K is given thickness of part (not modifiable!)

t is cross sectional length

- Modeling the stress at peg holes using convertion factor

$\sigma_{max} = K_t \sigma_A$ where K_t is proportional to $\left(\frac{\text{width of peg}}{t}\right)$

From part geometry → width of peg = .25 in

& Estimating t thickness → t = .5 in

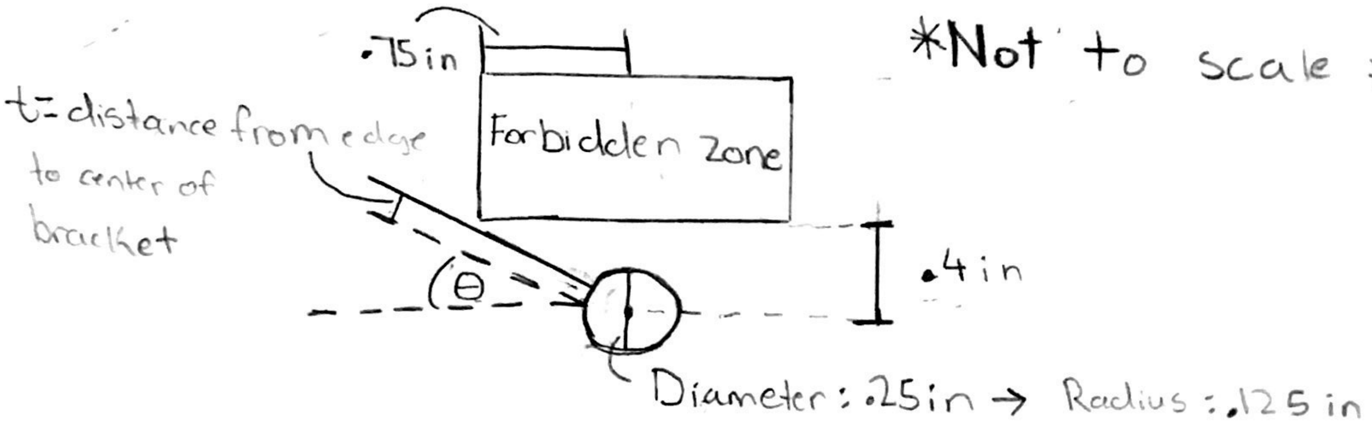
$\frac{.25}{.5} \rightarrow K_t \approx 2.2 = \frac{11}{5}$

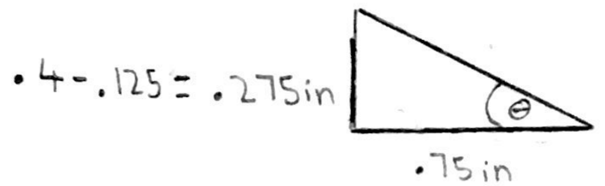
$$\sigma_{max} = \left(\frac{11}{5}\right)\left(\frac{mg\sqrt{1+\frac{w^2}{h^2}}}{2Kt}\right)$$

$$\boxed{\sigma_{max} = \frac{11\, mg\sqrt{1+\frac{w^2}{h^2}}}{10\, Kt}}$$

1) Analyzing Geometry

Not to scale





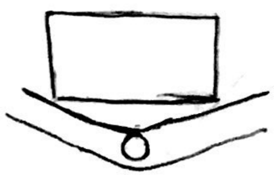
$$\theta = \tan^{-1}\left(\frac{.275}{.75}\right) = 20.136^\circ$$

Problem

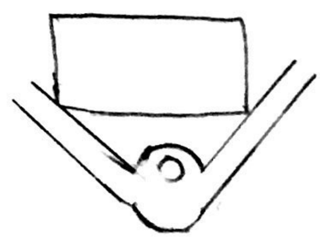
I want to maximize the amount of axial loading to avoid bending by increasing the angle, θ. However the forbidden Zone is in my way and the position of the hole can't be moved. If I reduce the amount material around the hole, its more likely to break due to smalll cross sectional area.

Idea

Instead of



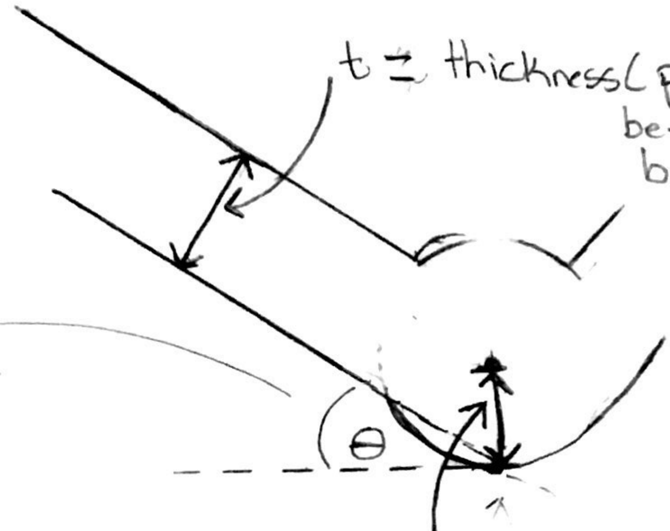
, I can

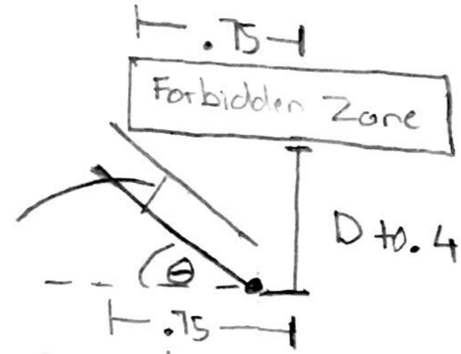


Advantages
- Keeps material around hole
- allows for a greater angle, θ

Disadvantages
- Reduce cross section area
- Sharp corner near the hole

2) Analyzing Geometry

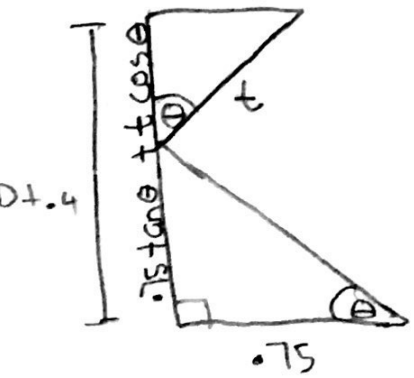




In order to avoid forbidden zone, the following formula must be satisfied:

$$\boxed{D + 0.4 > .75\tan\theta + t\cos\theta} \quad (1)$$

(Assuming D is constant, not a free variable)



3) Analyzing Geometry

Assuming that the angle θ is great enough that the bracket could be modeled as a set of pinned two force members as I analyzed earlier so only axial stress & no bending stress.

From earlier, Axial Force $F = \frac{mg}{2\sin\theta}$

Axial Stress $= \frac{F}{A} = \frac{mg}{2\sin\theta} \cdot \frac{1}{Kt} = \frac{mg}{2Kt\sin\theta}$

$K = .118$

$\sigma_{yield} = 10{,}000$ psi but I want a factor of safety of at least **5** so, max stress $= 2{,}000$ psi

$$\sigma_{max} = 2000 \geq \frac{mg}{2Kt\sin\theta}$$

$$\boxed{t \geq \frac{mg}{2\sigma_{max} K \sin\theta}} \quad (2)$$

Scanned with CamScanner

4)

Substituting Formula (2) into (1):

$$D + 0.4 > .75\tan\theta + \left(\frac{mg}{2\sigma_{max} K \sin\theta}\right)\left(\frac{\cos\theta}{1}\right)$$

$$D + 0.4 > .75\tan\theta + \frac{mg\cot\theta}{2\sigma_{max} K}$$

plugging in: $mg = 25\,lb$

$\sigma_{max} = 3{,}000$

$K = .118$

$$D > .75\tan\theta + .0529\cot\theta - .4$$

Ran FEA stress tests on

Excel sheet of different geometry values calculated:

	A	B	C	D	E
1	θ (degrees)	t(thickness of arm)	D (vertical distance from bottom edge to peg hole center)		Avoidance
2	30	0.105932203	0.124752681		0.324753
3	35	0.09234358	0.200799086		0.400799
4	40	0.082400626	0.292447265		0.492447
5	45	0.074905379	0.402966102		0.602966
6	50	0.069142335	0.538259031		0.738259
7	55	0.064659671	0.708198269		0.908198
8	60	0.061159986	0.929618099		1.129618
9					
10					
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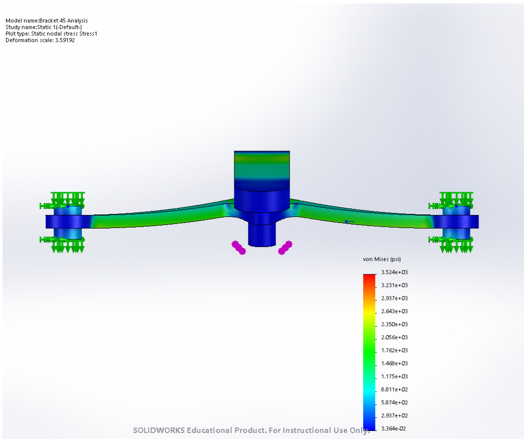
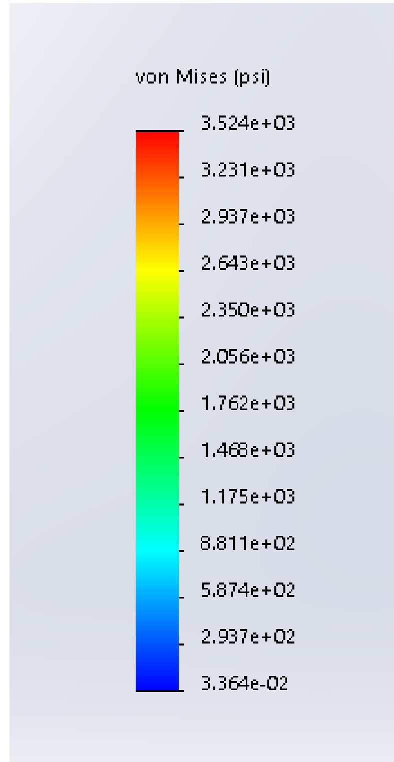
17					

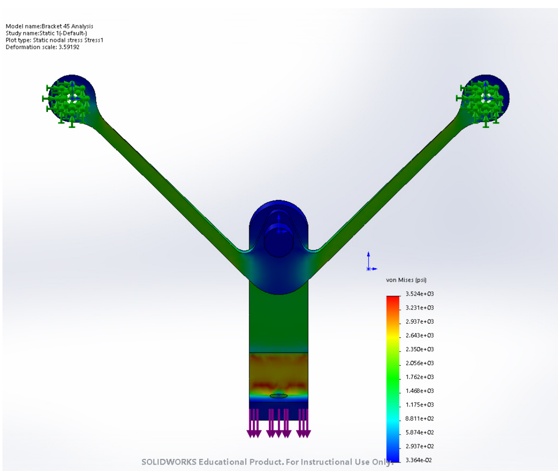
4. Detailed Modeling and Analysis of Final Design

Top:

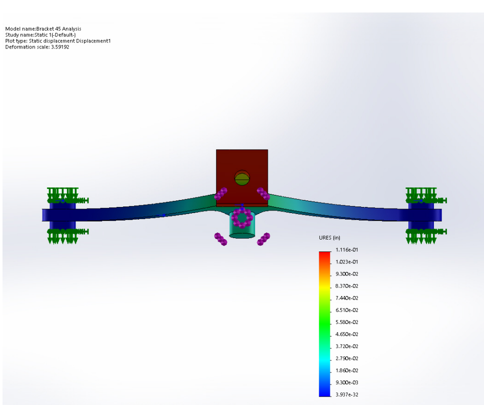


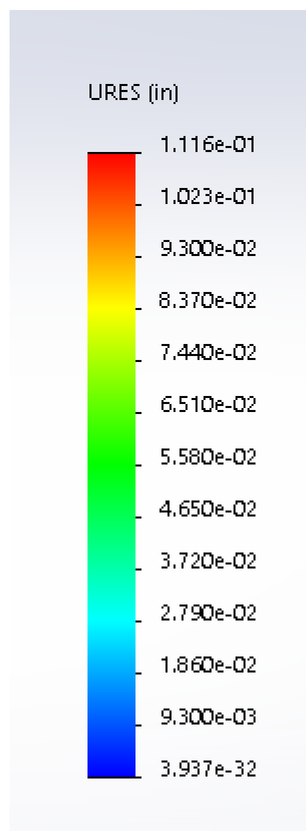


Middle:



Bottom:





4. Detailed Modeling and Analysis of Final Design (Con.)

My original design included a bracket with a 30 degree angle off the horizontal. Although it worked, in my FEA analysis I saw signs of bending stresses along the arms which I could not easily model in my simplified analysis. So in my new and final design, I wanted to increase the angle with the horizontal as much as possible to reduce bending stresses and increase axial stresses. I developed a formula which I derived in 3d which calculates the most optimized geometric variables for a given angle and F.o.S. I have also included an excel spreadsheet of optimized geometric calculations for angles between 30 and 60 degrees. I was only able to run FEA analysis on 3 angles, 30, 37, and 45 degrees.

When I ran my FEA analysis, the assumptions I made in my simplified analysis proved to be tremendously accurate in predicting stresses. By assuming the angle of the bracket was great enough that axial stresses were greater than bending stress (so I could model my bracket as 2 pinned two-force members), I was able to accurately predict actual stresses. This is evident by my F.o.S which I designed to be 5 and based on my FEA results was actually around 4.5 which is fairly close!

In reality, the angle I chose could have risen up to 60 degrees but as you'll see in my excel calculations, as the angle increases, the geometric parameters start to exponentially increase creating a design that can no longer be modeled as a 2 force member and for mass purposes is not feasible.

5. Manufacturing Report

Notes:
File attached is a pdf file.
Laser cutting Settings:

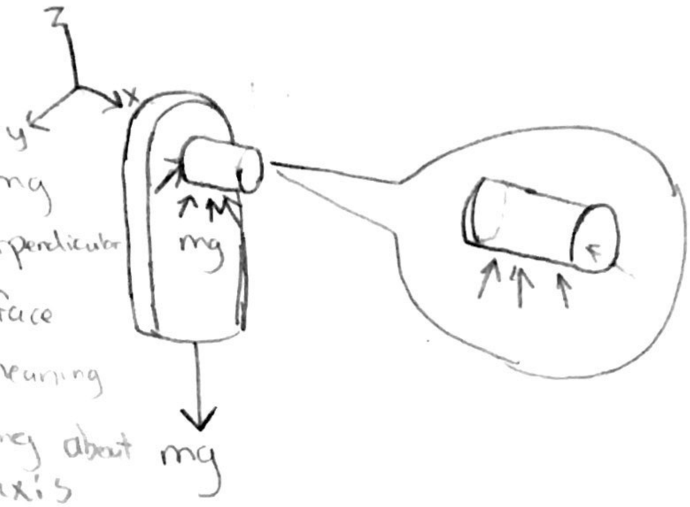
- Laser Printer: Epilog Mini #1:
- Speed: 10%
- Power: 90%
- Frequency: 5000 hz

Part contains numerous fillets of diameter 0.1in around center so maximum power should be 100.

6. (Supporting Notes)

(See Next Page)

Analysis on Support Clip



* Assume mg acts perpendicular to flat face of pin meaning no bending about y axis

Model as a rigidly connected pin with contact stresses

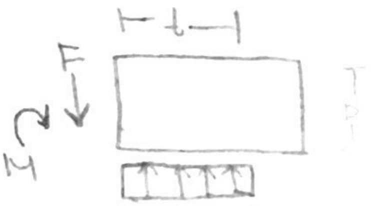
Parameters

t equals thickness of acrylic

D equals diameter of pin

FBD

Side view



mg (evenly distributed load)

$\Sigma F_y \rightarrow \quad F = mg$

$\Sigma M \rightarrow \quad M = \frac{mg\,t}{2}$

Stresses

$\text{Shear} = \tau = \frac{F}{A} = mg \frac{4}{\pi D^2} = \boxed{\frac{4mg}{\pi D^2}}$

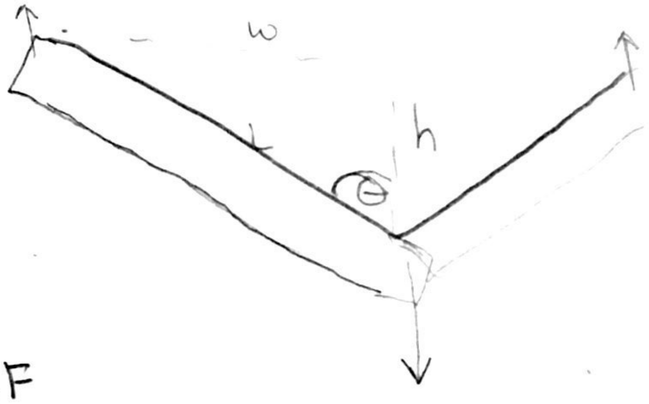
$\sigma_{bearing} = \boxed{\frac{mg}{Dt}}$

$\text{Bending} = \sigma_m = \frac{My}{I} = \frac{mg\,t}{2} \frac{D}{2} \frac{64}{\pi D^4} = \boxed{\frac{16\,mg\,t}{\pi D^3}}$

$\sigma_{bearing}$ will be the largest

First initial Brainstorm & Quick Calculations

where $L(w,h) = \sqrt{w^2+h^2}$ & $\Theta(w,h) = \tan^{-1}\left(\frac{w}{h}\right)$

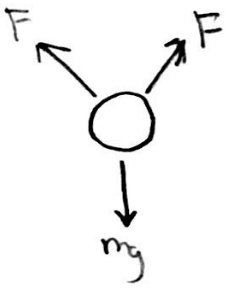


$\frac{w^2+h^2}{h^2}$

$\frac{w^2}{h^2}+1$

$\frac{w}{h}+1$

$L = \sqrt{w^2+h^2}$

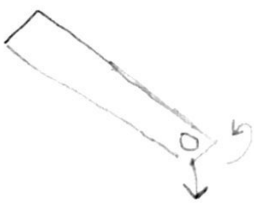


$\sin\theta = \frac{h}{L}$

$\Sigma F_y \rightarrow 0 = -mg + 2F\sin\theta$

$mg = 2F\frac{h}{L}$

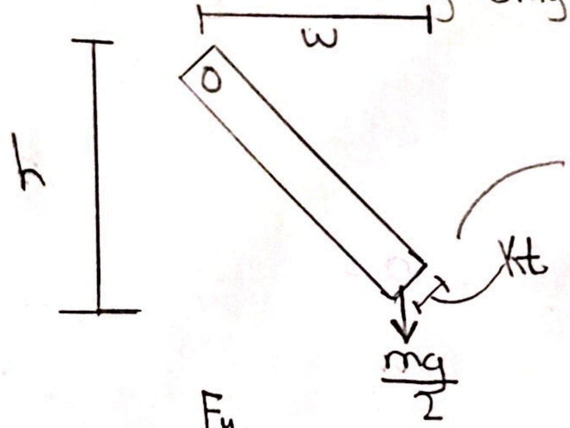
$F = \frac{mgL}{2h} = \frac{mg}{2}\left(\frac{w}{h}+1\right.$



a) continued

First analysis (Ended up being wrong)

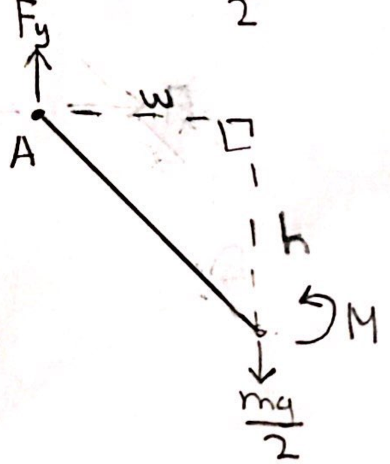
(Modeling only the left rect)



I cut the bracket internally which surfaced a moment and a force.

b)

(ignoring weight of part)



$$\Sigma F_y \rightarrow \boxed{F_y = \frac{mg}{2}}$$

$$\Sigma M|_A \rightarrow 0 = M - \frac{mgw}{2}$$

$$\boxed{M = \frac{mgw}{2}}$$

c)

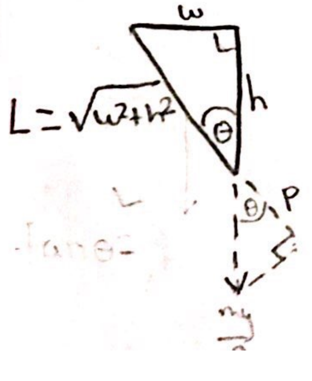
Possible Stresses

axial stress σ_A

Bending stress σ_m

Shear stress τ

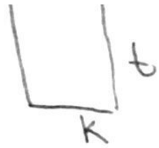
$$\sigma_A = \frac{\text{Force}}{\text{Area}} = \frac{P}{A} \quad \text{where } P = \frac{mg}{2}\cos\theta \text{ and } A = Kt$$



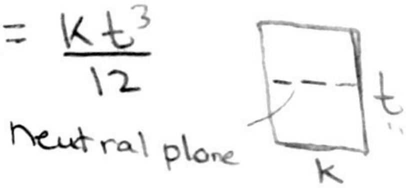
$$\boxed{\sigma_A = \frac{mg\cos\theta}{2Kt} \quad \text{where } \theta = \tan^{-1}\left(\frac{w}{h}\right)}$$

$$P = \frac{mg}{2}\cos\theta$$

$F = \frac{mg}{2}\sin\theta$, $A = Kt$ $\boxed{\tau = \frac{mg\sin\theta}{2Kt}}$



$\underline{\sigma_m} = \frac{My}{I}$ Where $M = \frac{mgw}{2}$, $y = \frac{t}{2}$, $I = \frac{Kt^3}{12}$



$$\sigma_m = \frac{mgw}{2} \cdot \frac{t}{2} \cdot \frac{Kt^3}{12}$$

$$\boxed{\sigma_m = \frac{mgwKt^4}{48}}$$

Comparing stresses

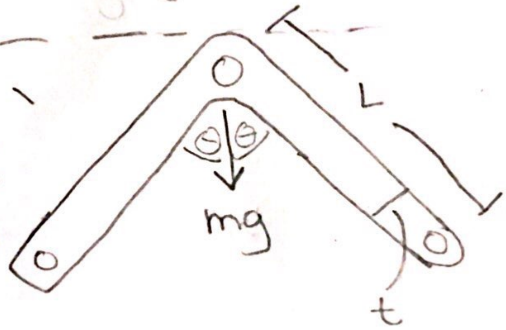
- $\frac{\tau}{\sigma_A} = \frac{\cancel{mg}\sin\theta}{\cancel{2Kt}} \cdot \frac{\cancel{2Kt}}{\cancel{mg}\cos\theta} = \tan\theta$

Based on geometry of the part and restrictions on part place $w > h$ and so $\tan\theta = \frac{w}{h} > 1$ so τ, shear stress $>$ σ_A, axial stress

- $\frac{\sigma_m}{\tau} = \frac{\cancel{mg}wKt^4}{48} \cdot \frac{2Kt}{\cancel{mg}\sin\theta} = \frac{wK^2t^5}{48\sin\theta}$

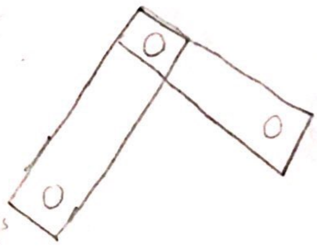
$\sin\theta = \frac{w}{\sqrt{w^2+h^2}} \rightarrow \frac{\cancel{w}K^2t^5}{48} \cdot \frac{\sqrt{w^2+h^2}}{\cancel{w}} = \frac{K^2t^5\sqrt{w^2+h^2}}{48}$

Other Possible Designs (i)



Possible advantages

- Don't have to worry about restricted zone
- predominantly axial loading



Possible disadvantages

- bracket is in compression → Buckling

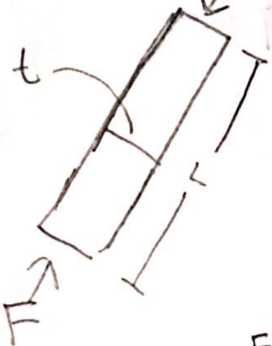
where — θ = angle of bracket sides from vertical axis

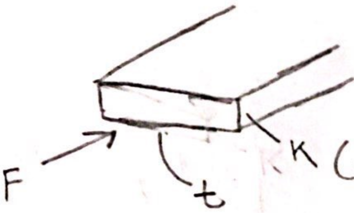
- L is length of a side
- t is width of bracket

Like my current design, treat it as rectangles connected at a pin. Allows the system to be modeled as 2 symmetric two-force members

FBD of pin

$$\Sigma F_y \rightarrow F = \frac{mg}{2\cos\theta}$$





K (thickness of acrylic

Stresses

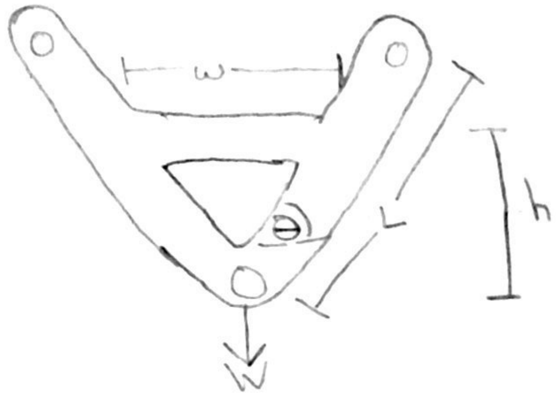
- axial = $\sigma = \frac{F}{A} = \left(\frac{mg}{2\cos\theta}\right)\left(\frac{1}{Kt}\right) = \frac{mg}{2Kt\cos\theta}$
- buckling $F_{cr} = \frac{\pi^2 EI}{4L^2} = \frac{\pi^2 EtK^3}{48L^2}$

Not a useful design because bracket is in compression which creates the possibility of buckling

Other Designs (2)

Possible Advantages

- Strut across improves rigidity and helps reduce bending



Parameters

h = height of strut from support clip

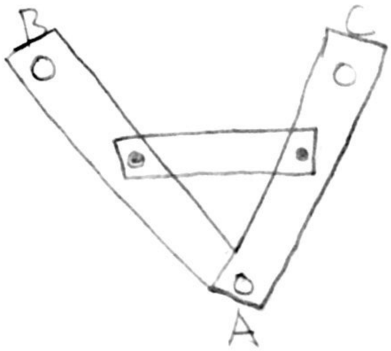
w = width of strut

θ = angle of arms measured from horizontal

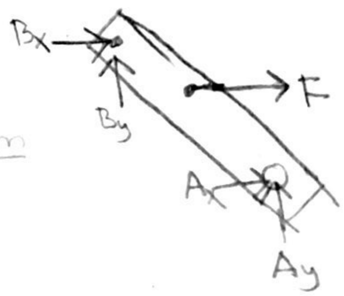
L = length of arm

Disadvantages

- Can't simply model the bracket as a set of two force members
- Added mass
- Generally harder to predict stresses without FEA
- Many more parameters than V shape



left arm



Statically indeterminate

Not simple enough

Strut

Interesting idea for reducing bending but should be solved using FEA due to complexity